

AP Calculus Summer Review Packet

This is all STUFF YOU SHOULD KNOW. We will not be reviewing precal or trig material.

This will not be turned in for a grade.

You are welcome to email me with questions: khicks@madison-schools.com.

There will be a test on the 2nd day of class covering this material.

Topic A: Functions

1.) If $f(x) = 4x - x^2$, find:

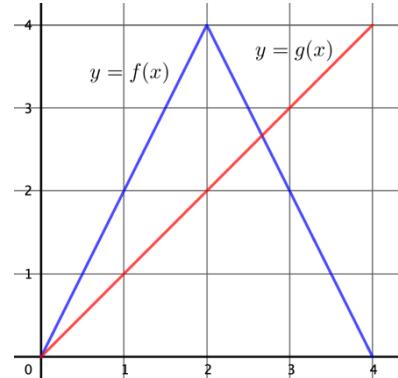
a.) $f(4) - f(-4)$

b.) $\frac{f(x+h) - f(x)}{h}$

2.) If $f(x)$ and $g(x)$ are given in the graph, find:

a.) $(f - g)(3)$

b.) $f(g(3))$



3.) If $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \leq x < 2 \\ \sqrt{x+2} - 2, & x \geq 2 \end{cases}$, find:

a.) $f(0) - f(2)$

b.) $f(f(3))$

Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.) $f(x) = 3$

2.) $y = x^3 - x^2 + x$

3.) $y = \frac{x^3 - x^2 + x}{x}$

4.) $y = \frac{x-4}{x^2 - 16}$

5.) $f(x) = \frac{1}{4x^2 - 4x - 3}$

6.) $y = \sqrt{2x - 9}$

7.) $y = \log(x - 10)$

8.) $y = \frac{\sqrt{2x + 14}}{x^2 - 49}$

Find the range of the following functions:

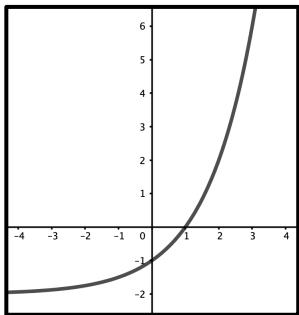
9.) $y = x^4 + x^2 - 1$

10.) $y = 100^x$

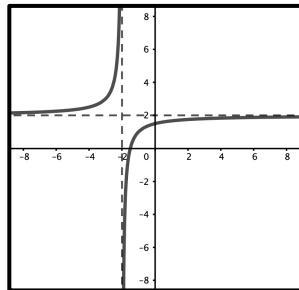
11.) $y = \sqrt{x^2 + 1} + 1$

Find the domain and range of the following functions using interval notation.

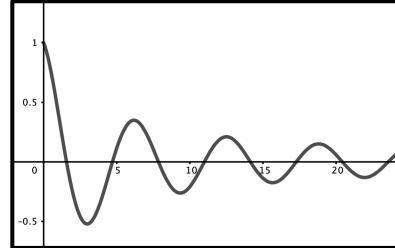
12.)



13.)



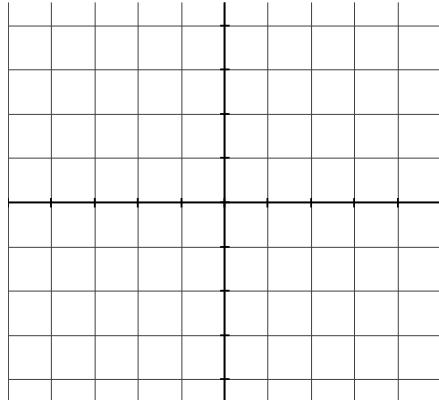
14.)



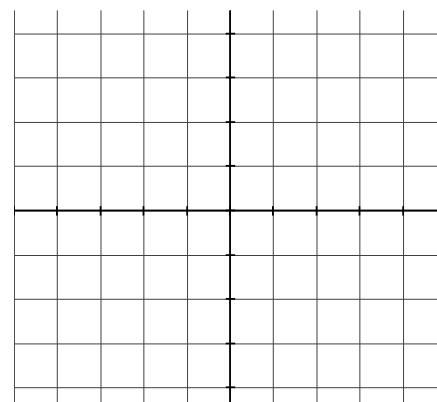
Topic C: Graphs of Common Functions

Sketch each of the following as accurately as possible. **You will need to be VERY familiar with each of these graphs throughout the year.** You may use a graphing calculator or www.desmos.com/calculator. There is an app for Desmos as well that is free that you can install on your phones. Again, these are VERY important graphs to know. Be very accurate with regards to “open circles” and “closed circles” as those features may not be revealed on a graphing utility.

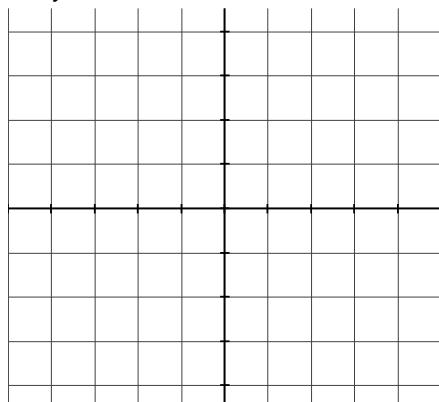
1. $y = x$



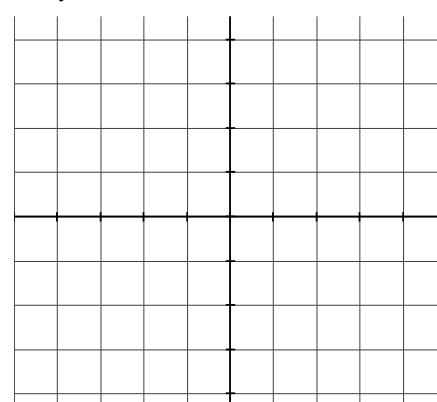
2. $y = x^2$



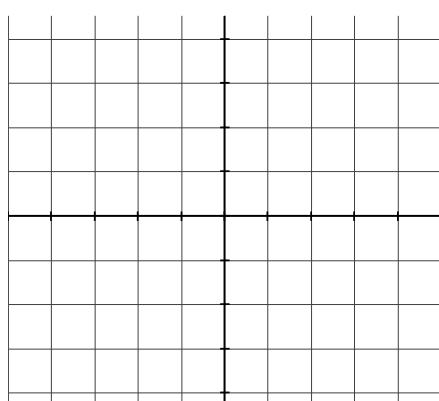
3. $y = x^3$



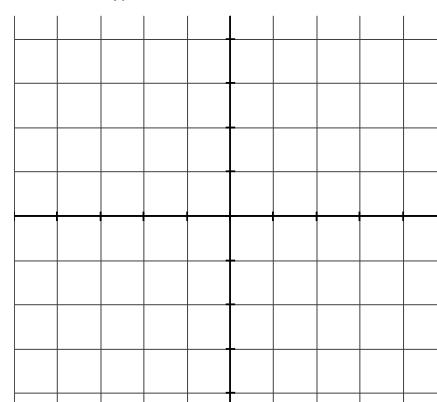
4. $y = \sqrt{x}$



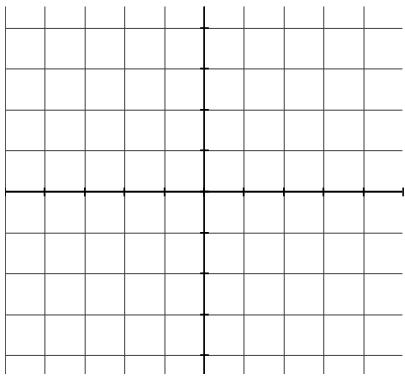
5. $y = |x|$



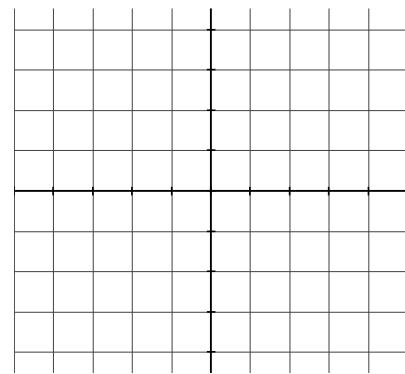
6. $y = \frac{|x|}{x}$



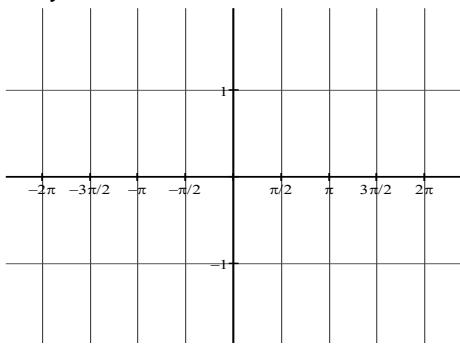
7. $y = x^{1/3}$



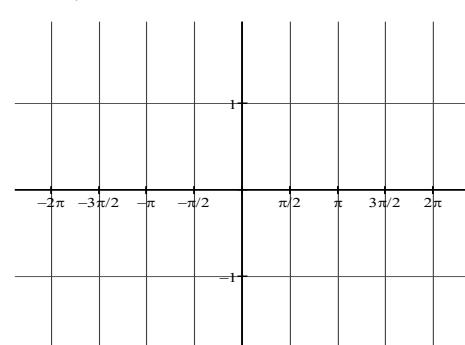
8. $y = x^{2/3}$



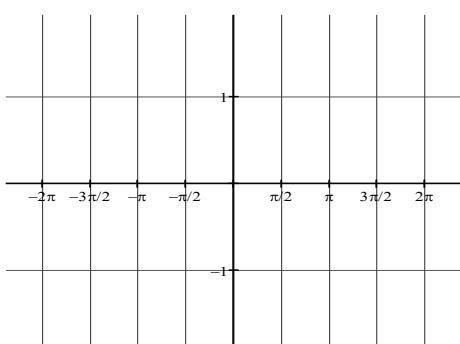
9. $y = \sin x$



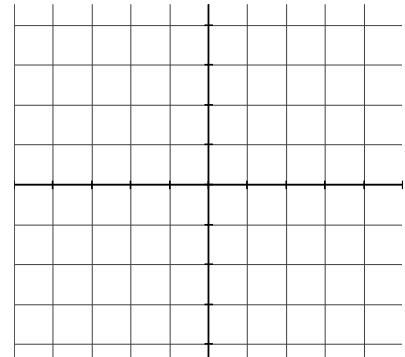
10. $y = \cos x$



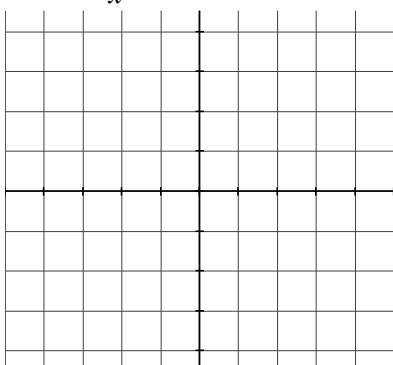
11. $y = \tan x$



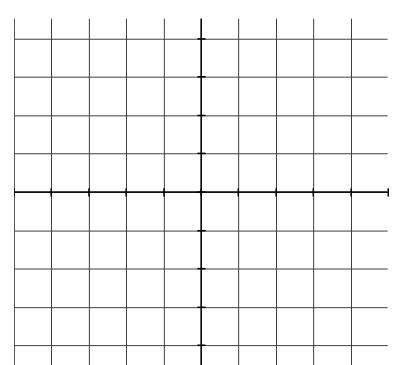
12. $y = e^x$



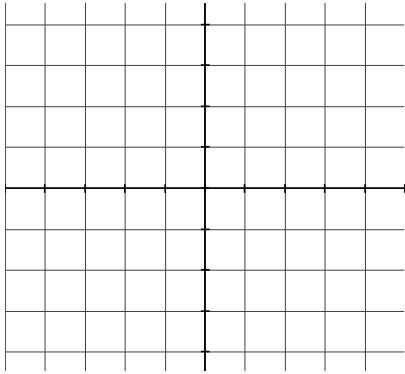
13. $y = \frac{1}{x}$



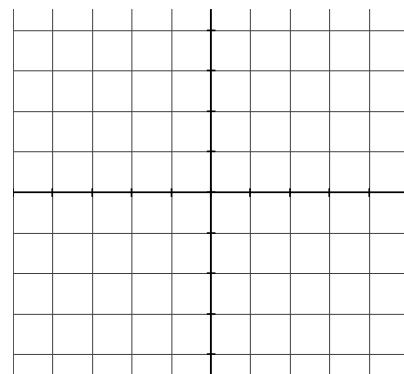
14. $y = \ln x$



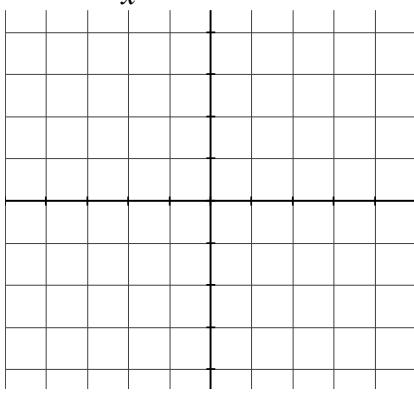
15. $y = \sqrt{x^2 + 9}$



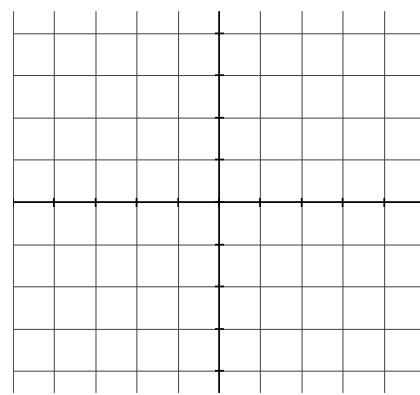
16. $y = \sqrt{4 - x^2}$



17. $y = \frac{1}{x^2}$



18. $y = 2^x$



Topic D: Function Transformations

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1.) $f(x) - 4$

2.) $f(x - 4)$

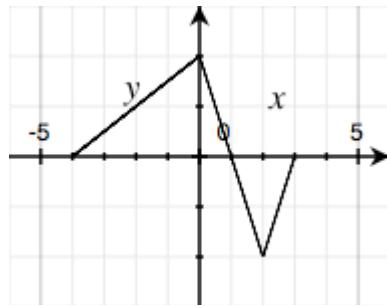
3.) $-f(x + 2)$

4.) $5f(x) + 3$

5.) $f(2x)$

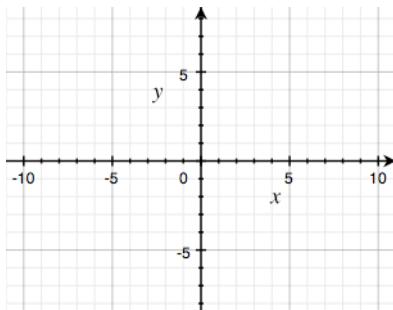
6.) $|f(x)|$

Here is a graph of $y = f(x)$:

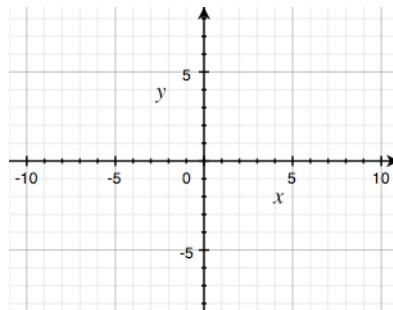


Sketch the following graphs:

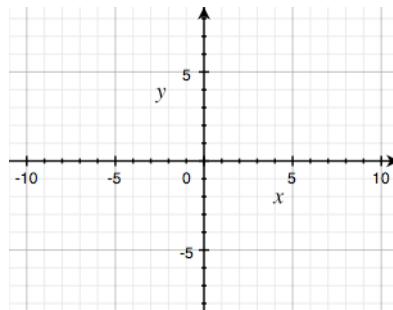
7.) $y = 2f(x)$



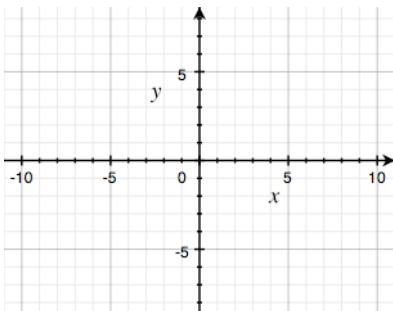
8.) $y = -f(x)$



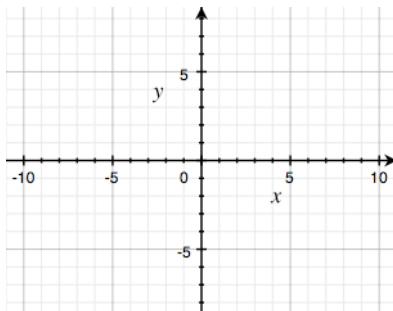
9.) $y = f(x - 1)$



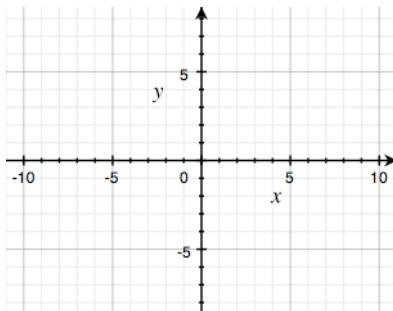
10.) $y = f(x) + 2$



11.) $y = |f(x)|$



12.) $y = f(|x|)$



Topic E: Special Factorization

Factor completely.

1.) $x^3 + 8$

2.) $x^3 - 8$

3.) $27x^3 - 125y^3$

4.) $x^4 + 11x^2 - 80$

5.) $ac + cd - ab - bd$

6.) $2x^2 + 50y^2 - 20xy$

7.) $x^2 + 12x + 36 - 9y^2$

8.) $x^3 - xy^2 + x^2y - y^3$

9.) $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

* I don't know why, but SO MANY of my students struggle with factoring. You MUST KNOW how to factor a trinomial.

Topic F: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m = -7, (-3, -7)$

b.) $m = -\frac{1}{2}, (2, -8)$

2.) Find the equation of the line in point-slope form, passing through the given points.

a.) $(-3, 6), (-1, 2)$

b.) $\left(-2, \frac{2}{3}\right), \left(\frac{1}{2}, 1\right)$

3.) Find the equations of the lines through the given point that are a.) parallel and b.) normal to the given line.

a.)

b.) $(-6, 2), 5x + 2y = 7$

4.) Find the equation of the line in general form, containing the point $(4, -2)$ and parallel to the line containing the points $(-1, 4)$ and $(2, 3)$.

5.) Find k if the lines $3x - 5y = 9$ and $2x + ky = 11$ are a.) parallel and b.) perpendicular.

Topic G: Solving Quadratic and Polynomial Equations

Solve each equation for x over the real number system.

$$1.) \ x^2 + 7x - 18 = 0$$

$$2.) \ x^2 + x + \frac{1}{4} = 0$$

$$3.) \ 2x^2 - 72 = 0$$

$$4.) \ 20x^2 - 56x + 15 = 0$$

$$5.) \ 81x^2 + 72x + 16 = 0$$

$$6.) \ x + \frac{1}{x} = \frac{17}{4}$$

*Again – you MUST know how to factor a trinomial.

Topic H: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

$$1.) \ y = \frac{x-1}{x+5}$$

$$2.) \ y = \frac{2x+16}{x+8}$$

$$3.) \ y = \frac{x}{x^2 - 25}$$

$$4.) \ y = \frac{x^2 - 5}{2x^2 - 12}$$

$$5.) \ y = \frac{x^3}{x^2 + 4}$$

$$6.) \ y = \frac{10x + 20}{x^3 - 2x^2 - 4x + 8}$$

$$7.) \ y = \frac{1}{x} - \frac{x}{x+2} \text{ (Hint: Express with a common denominator)}$$

Topic I: Negative and Fractional Exponents

Simplify and write with positive exponents.

$$1.) -12^2 x^{-5}$$

$$2.) (-12x^5)^{-2}$$

$$3.) (4x^{-1})^{-1}$$

$$4.) \left(\frac{-4}{x^4}\right)^{-3}$$

$$5.) \left(\frac{5x^3}{y^2}\right)^{-3}$$

$$6.) (8x^2)^{-\frac{4}{3}}$$

$$7.) \frac{(x^2-1)^{-\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}}$$

$$8.) (x^{-2} + 2^{-2})^{-1}$$

Topic J: Complex Fractions

Eliminate the complex fractions:

$$1.) \frac{\frac{5}{8}}{-\frac{2}{3}}$$

$$2.) \frac{4 - \frac{2}{9}}{3 + \frac{4}{3}}$$

$$3.) \frac{2 + \frac{7}{2} + \frac{3}{5}}{5 - \frac{3}{4}}$$

$$4.) \frac{x - \frac{1}{x}}{x + \frac{1}{x}}$$

$$5.) \frac{x^{-2} + x^{-1} + 1}{x^{-2} - x}$$

$$6.) \frac{2x(2x-1)^{\frac{1}{2}} - 2x^2(2x-1)^{-\frac{1}{2}}}{(2x-1)}$$

Topic K: Inverses

Find the inverse of each of the following functions.

$$1.) \ 2x - 6y = 1$$

$$2.) \ y = 9 - x^2, x \geq 0$$

$$3.) \ y = \sqrt{1 - x^3}$$

$$4.) \ y = \frac{9}{x}$$

Find the inverse of each of the following functions and show that $f(f^{-1}(x)) = x$

$$5.) \ f(x) = \frac{1}{2}x - \frac{4}{5}$$

$$6.) \ f(x) = x^2 - 4$$

$$7.) \text{ Without finding the inverse, find the domain and range of the inverse to } f(x) = \frac{\sqrt{x+1}}{x^2}$$

Topic L: Adding Fractions and Solving Rational Equations

1.) Combine the following fractions:

a.) $\frac{2}{3} - \frac{1}{x}$

b.) $\frac{1}{x-3} + \frac{1}{x+3}$

c.) $\frac{5}{2x} - \frac{5}{3x+15}$

d.) $\frac{2x-1}{x-1} - \frac{3x}{2x+1}$

2.) Solve the equation for x .

a.) $\frac{2}{3} - \frac{1}{x} = \frac{5}{6}$

b.) $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$

c.) $\frac{5}{2x} - \frac{5}{3(x+5)} = \frac{5}{x}$

d.) $\frac{2x-1}{x-1} - \frac{3x}{2x+1} = \frac{x^2+11}{2x^2-x-1}$

Topic M: Absolute Value Equations

Solve the following equations:

$$1.) |1 - 7x| = 13$$

$$2.) |8 + 2x| + 2x = 40$$

$$3.) |4x - 5| + 5x + 2 = 0$$

$$4.) |x^2 - 2x - 1| = 7$$

Topic N: Solving Inequalities

Solve the following inequalities:

$$1.) 5(x - 3) \leq 8(x + 5)$$

$$2.) 4 - \frac{5x}{3} > -\left(2x + \frac{1}{2}\right)$$

$$3.) \frac{3}{4} > x + 1 > \frac{1}{2}$$

$$4.) \frac{5}{x - 6} \geq \frac{1}{x + 2}$$

Topic O: Exponential Functions and Logarithms

Simplify the following:

$$1.) \log_2 \frac{1}{4}$$

$$2.) \log_8 4$$

$$3.) \ln \frac{1}{\sqrt[3]{e^2}}$$

$$4.) 5^{\log_5 40}$$

$$5.) e^{\ln 12}$$

$$6.) 3 \ln(x - 1) + \frac{1}{2} \ln(x + 3) - \ln(6x)$$

Solve the following:

$$7.) \log_2(x - 1) + \log_2(x + 3) = 5$$

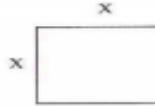
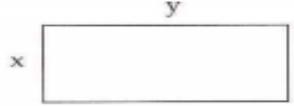
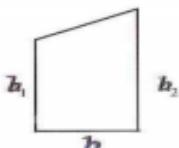
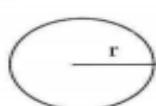
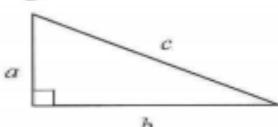
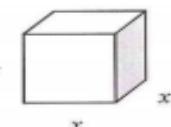
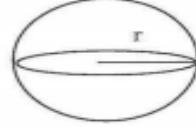
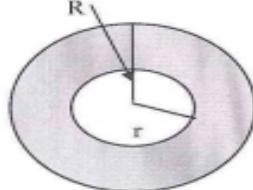
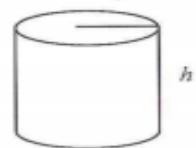
$$8.) \ln x^3 - \ln x^2 = \frac{1}{2}$$

$$9.) e^{3x+1} = 10$$

$$10.) 8^x = 5^{2x-1}$$

Topic P: Geometry

1.) You will use each of the following formulas in AP Calculus AB. Complete each of the following.

Square  Perimeter = _____ Area = _____	Rectangle  Perimeter = _____ Area = _____	Trapezoid  Area = _____
Circle  Circumference = _____ Area = _____	Triangle  Pythagorean Theorem (only good for right triangles) = _____ Area (of any triangle) = _____	Cube  Volume = _____ Surface Area = _____
Sphere  Volume = _____	"Washer"  Area of the shaded region = _____	Cylinder  Volume = _____

Topic Q: Basic Right-Angle Trigonometry

Solve the following:

If point P is on the terminal side of θ , find all 6 trigonometric functions of θ . (Answers need not be rationalized.)

1.) $P(-2,4)$

2.) $P(\sqrt{5}, -2)$

3.) If $\cos\theta = -\frac{5}{13}$, in quadrant II,
find $\sin\theta$ and $\tan\theta$.

4.) If $\cot\theta = \frac{2\sqrt{10}}{3}$, in quadrant III,
find $\sin\theta$ and $\cos\theta$.

5.) State the quadrant in which each of the following is true.

a.) $\sin\theta > 0$ and $\cos\theta < 0$

b.) $\tan\theta > 0$ and $\sec\theta < 0$

Topic R: Special Angles

Evaluate each of the following.

$$1.) \sin^2 120^\circ + \cos^2 120^\circ$$

$$2.) 2\tan^2 300^\circ + 3\sin^2 150^\circ - \cos^2 180^\circ$$

$$3.) \cot^2 135^\circ - \sin^2 210^\circ + 5\cos^2 225^\circ$$

$$4.) \cot(-30^\circ) + 3\tan 600^\circ - \csc(-450^\circ)$$

Topic S: Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$. Do not use a calculator.

$$1.) \sin^2 x = \sin x$$

$$2.) 3\tan^3 x = \tan x$$

$$3.) \sin^2 x = 3\cos^2 x$$

$$4.) \cos x + \sin x \tan x = 2$$

$$5.) \sin x = \cos x$$

$$6.) 2\cos^2 x + \sin x - 1 = 0$$